Mathematical Programming Glossary Supplement: A Degenerate Polyhedron

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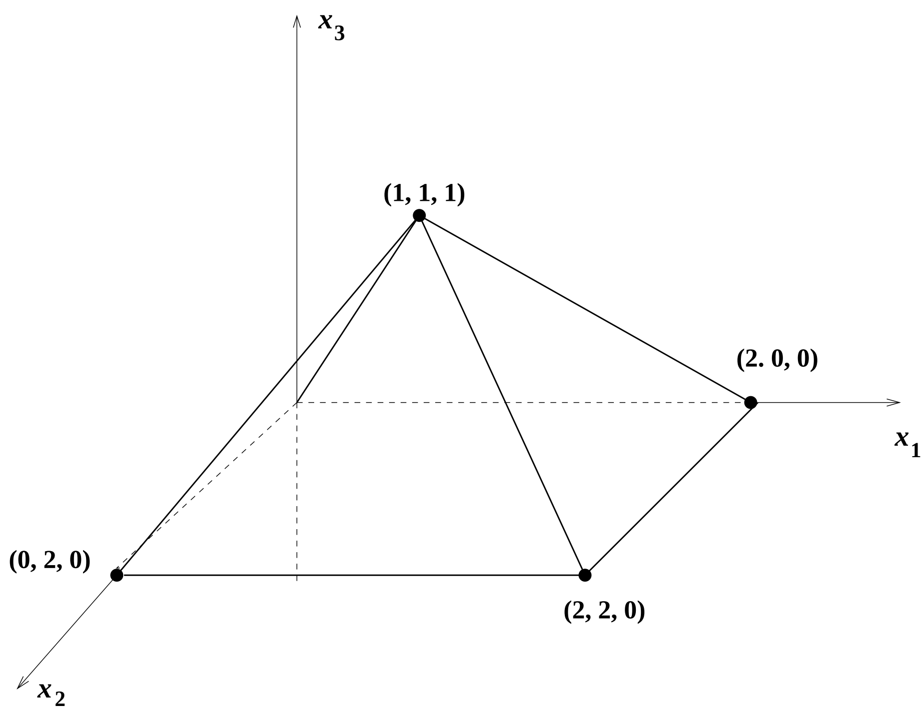
(url changed December 1, 1998)

June 27, 1996

The following pyramid is degenerate: the point $(1, 1, 1)$ is the intersection of four faces, so it is a degenerate extreme point. (Note that no face is redundant.)



$$
\begin{array}{rrrrrl}
 & x_1 & & + & x_3 & \le 2 \\
- & x_1 & & + & x_3 & \le 0 \\
 & & x_2 & + & x_3 & \le 2 \\
 & - & x_2 & + & x_3 & \le 0 \\
\multicolumn{6}{c}{x_1, x_2, x_3 \ge 0.}
\end{array}
$$

From a linear programming view, put the system into standard form:

$$
\begin{array}{rrrrrrrrrl}
 & x_1 & & + x_3 & + x_4 & & & & = 2 \\
- & x_1 & & + x_3 & & + x_5 & & & = 0 \\
 & & x_2 & + x_3 & & & + x_6 & & = 2 \\
 & - & x_2 & + x_3 & & & & + x_7 & = 0
\end{array}
$$

$$x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0.$$

The three bases that correspond to the one extreme point are given by the following tableaux:

Basic	*Level*	x_5	x_6	x_7
x_1	1	-1	$\frac{1}{2}$	$\frac{1}{2}$
x_2	1	0	$\frac{1}{2}$	$-\frac{1}{2}$
x_3	1	0	$\frac{1}{2}$	$\frac{1}{2}$
x_4	0	1	-1	-1

Basic	*Level*	x_4	x_6	x_7
x_1	1	1	$-\frac{1}{2}$	$-\frac{1}{2}$
x_2	1	0	$\frac{1}{2}$	$-\frac{1}{2}$
x_3	1	0	$\frac{1}{2}$	$\frac{1}{2}$
x_5	0	1	-1	-1

Basic	*Level*	x_4	x_5	x_7
x_1	1	$\frac{1}{2}$	$-\frac{1}{2}$	0
x_2	1	$\frac{1}{2}$	$\frac{1}{2}$	-1
x_3	1	$\frac{1}{2}$	$\frac{1}{2}$	0
x_6	0	-1	-1	1

Basic	*Level*	x_4	x_5	x_6
x_1	1	$\frac{1}{2}$	$-\frac{1}{2}$	0
x_2	1	$-\frac{1}{2}$	$-\frac{1}{2}$	1
x_3	1	$\frac{1}{2}$	$\frac{1}{2}$	0
x_7	0	-1	-1	1